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Configuration model of partial repairable spares under batch ordering policy based on inventory state

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Abstract Rational planning of spares configuration project is an effective approach to improve equipment availability as well as reduce life cycle cost (LCC). With an analysis of various impacts on support system, the spares demand rate forecast model is constructed. According to systemic analysis method, spares support effectiveness evaluation indicators system is built, and then, initial spares configuration and optimization method is researched. To the issue of discarding and consumption for incomplete repairable items, its expected backorders function is approximated by Laplace demand distribution. Combining the ($s-1$, s) and (R, Q) inventory policy, the spares resupply model is established under the batch ordering policy based on inventory state, and the optimization analysis flow for spares configuration is proposed. Through application on shipborne equipment spares configuration, the given scenarios are analyzed under two constraint targets: one is the support effectiveness, and the other is the spares cost. Analysis reveals that the result is consistent with practical regulation; therefore, the model's correctness, method's validity as well as optimization project's rationality are proved to a certain extent.

1. Introduction

Spares, which are closely accompanied with the whole life cycle of equipment, are supportability material for equipment maintenance and emergency repair and are also the material basis for equipment support and supply,[1] for it directly impacts the life cycle cost (LCC) as well as the equipment readiness posture. Currently, military mainly rely on "excessive procurement and reserves" to meet the spares support demand, which leads to a large amount of spares stock putting off. In this case, spares may be damage or failure during storage and cause a lot of waste; in addition, the actual needed spares are of serious shortage, which seriously affect the equipment support effectiveness.

The above-mentioned issue involves the spares configuration and optimization. Firstly, spares need to be classified as repairable or consumable items so as to determine the reasonable inventory strategy.[2] For repairable items of high failure rate, low consumption and expensive cost, ($s-1$, s) inventory policy is introduced, which is the basic assumption for multi-echelon technique for recoverable item control (METRIC). Under this assumption, Cesaro and Pacciarelli[3] and Lee et al.[4]

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researched the issues of aviation spares optimization; Sleptchenko[5] and Rustenburg et al.[6] proposed spares configuration method for naval warship based on METRIC; Ruan et al.[7] established three-echelon inventory optimization model of repairable spares, and proposed the concept of system support degree; in addition, he researched the optimization method of carrying spares support project for warship equipment under multi-constraints.[8] Diaz and Fu[9] and Ruiz-Castro and Li[10] relaxed METRIC assumptions of "unlimited maintenance overall", and researched spares optimization method under the maintenance resource constraints. For non-repairable spares of low failure rate and high consumption, (R, Q) inventory policy is usually applied,[11] that is, determine reasonable ordering point R and economic ordering quantity Q based on spares stock state. Gumus and Guneri[12] studied inventory management system framework under the random demand and fuzzy supply chain environment; Topan et al.[13] and Darwish and Odah[14] researched the economic ordering policy of two-echelon inventory system that consists of a central warehouse and several point-of-sale; Olsson[15] and Tiacci and Saetta[16] studied the optimal inventory policy of consumable spares with lateral transshipment; Al-Rifai and Rossetti[17] built heuristic optimization model of two-echelon ordering system under the (R, Q) inventory policy, Mao et al.[18] studied the consumable spares optimization methods centered equipment availability.

For military equipment, most of the spares are partial repairable items, that is, the increase of repairing times will lead to spares performance degradation. But in the majority of the related literature, repairable items are always assumed to be repaired, and after each repair, it will recover the initial state; some actual cases and impact factors, such as spares service life and its discarding consumption, are not considered. In addition, periodically ordering or planned procurement from external supplier, spares project will appear to be poor robustness, and it is difficult to meet the randomness and uncertainty phenomenon of spares failure and consumption. In this paper, we will focus on incomplete recoverable equipment and take into account the factors that items performance become degradation after repair as well as its discarding and consumption. Under the batch ordering mode based on inventory state, we establish an optimal configuration model of partial repairable items through organic combination of $(s-1, s)$ and (R, Q) inventory policy, so as to make the spares project more reasonable and the calculation result close to the actual situation.

2. Description of spares support process

Spare parts, according to its indenture, can be classified into line replaceable units (LRU) and shop replaceable units (SRU).[19] If equipment fails in grass-root station, then detect and locate the fault, disassemble the failure item and install the spare in equipment. The process is shown in Fig. 1. If there are spares in stock, we can replace the failure item by its spares, and then the failure equipment can be recovered. If there are no spares in stock, one time of LRU shortage will occur. Considering the repair capacity restrictions, the failure item has certain repair probability in grass-root station; after repair, the recovered item will be stored in grass-root station and can be used for the next demand. If the failure item cannot be repaired, it will be sent to repair at higher echelon location (intermediate station); at the same time, the grass-root depot orders one spare from intermediate depot. The spares support process at intermediate station is similar to that of grass-root station.

According to equipment fault tree structure, the failure LRU is caused by its sub-assembly SRU. If there are SRU spares in stock, it will be installed in LRU to replace the failure SRU. Then, we complete the repair of the failure LRU. If there are no SRU spares in stock, it will delay the repair time. When we complete the repair and supply of failure LRU, a spare shortage is resolved.

In support system, the top-echelon site (base station) is usually constituted by industrial factory or military region repair shop and equipped with complete repair device, tools, and technical document; it has a strong repair capacity and can undertake all of the repair work. Actually, apart from the whole-life item, most of repairable parts have a certain service life; besides, its working performance will decline with the increase of their repair times, so the failure item may has a certain discarding rate at the top-echelon site. With the spares' discarding and consumption, the base station needs to purchase spares from external supplier based on the current inventory state. Considering the long supply cycle of some items (such as special, imported, or temporary production items, etc.), as well as the order fees and transportation costs, the base station needs to take batch ordering policy, thus, the amount of spares stock in the whole support system can meet the needs of equipment repair.

3. Spares demand rate

3.1. Analysis of spares demand

In order to quickly restore the combat effectiveness of failed equipment, the repair mode that replaces the failure items by its spares is usually used, in the way that spares demand rate equals its replacement rate, which is an important input parameter for spares optimization model. The main factors

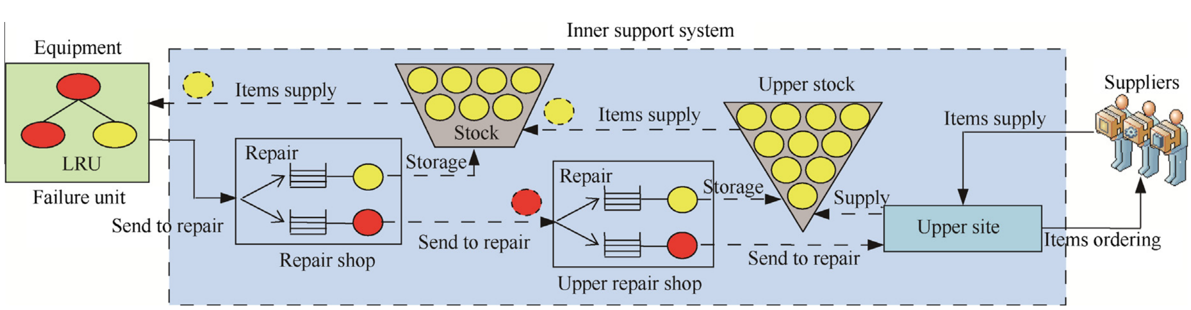

Fig. 1 Process of failure, repair, ordering and supply for spare parts.

Table 1 Influencing factors of spares demand rate.

Influencing factor	Corresponding model parameter
Equipment reliability	Average time between failures
Maintenance and support conditions	Repair in place rate
	Retest OK rate
	Repair probability
Equipment task intensity	Working time in a given interval
	Duty cycle of equipment components
System structure	Spares indenture in the system structure
	Items installation number in its higher indenture mother component
Equipment deployment	Deployment number of equipment in support system
Support organizational structure	Support site echelon
	Number of sites in support system
	Support and supply relations

to affect spares demand rate include equipment reliability, maintenance and repair conditions, repair capacity, equipment task intensity, system structure, equipment deployment and the support organizational structure. The factors influencing spares demand rate and its corresponding model parameters are given in Table 1.

During the early stages of new equipment in service, there is no historical failure and consumption data and the equipment design targets of reliability, maintenance and supportability (RMS) shown in Table 1 can be used to forecast spares demand rate.

3.2. Spares demand rate calculation model

Suppose m = 1,2,..., M, is the support site index, j = 1,2,..., J, is the item index. The analysis of spares maintenance supply process shows that the arrival rate λ_{mj} of item j at support site m constitutes two parts: the one is the total amount of item j that cannot be repaired at the site m's downstream sites l ($l \in \text{Unit}(m)$), the other is the demand rates of item j at site m when repairing its higher indenture items (i.e., assembly l have item j as subassembly, $l \in \text{Aub}(j)$). Hence, spares demand rate is the sum of the two parts described above.

$$\lambda_{mj} = \sum_{l\in\text{Unit}(m)} \lambda_{lj}\text{NRTS}_{lj} + \sum_{l\in\text{Aub}(j)} \lambda_{ml}(1-\text{NRTS}_{ml})q_{mlj}R_{mj} \qquad (1)$$

where Unit(m) expresses the downstream of site m, NRTS_{lj} is the probability that item j cannot be repaired at location l ($l \in \text{Unit}(m)$), NRTS_{ml} the probability that item l ($l \in \text{Aub}(j)$, which expresses the maternal component of item j) cannot be repaired at location m, q_{mlj} the fault isolation probability, that is, the probability that failure item l at location m is caused by its subassembly j and R_{mj} indicates the replaceable probability of item j at location m. If $R_{mj} = 1$, the failure item j can be always replaced at location m; if $R_{mj} = 0$, we can assume that the failure item j cannot be replaced at location m.

According to the equipment deployment amount, system structure and repair conditions, we can get the LRU's annual demand rate λ_{mj} at the first echelon site.[20]

$$\lambda_{mj} = \frac{365\text{DC}_j(1-\text{RIP}_j)\text{HW}_m Z_j N_m}{7\text{MTBF}_j(1-\text{RtOK}_j)} \qquad (2)$$
$$m \in \text{Echelon}(N),\ j \in \text{Inden}(1)$$

where DC_j is duty cycle, RIP_j the repair in place rate, HW_m the mean working time (hours) of each week, Z_j the install number of item j in its higher indenture component, N_m the equipment deployment amount at site m, MTBF_j the average time between failures of the jth item, and RtOK_j the retest OK rate for item j. Echelon(N) represents the lowest echelon site and Inden(1) the first indenture item LRU.

The fault isolation probability q_{mjk} can be expressed as

$$q_{mjk} = \frac{\text{DC}_k Z_k \text{MTBF}_j(1-\text{RtOK}_j)(1-\text{RIP}_k)}{\text{MTBF}_k(1-\text{RtOK}_k)(1-\text{RIP}_j)} \qquad (3)$$

The subscript k represents the subassembly of item j, that is to say, item j is the maternal component of item k.

4. Initial spares configuration model

For new equipment in service, the early support period is important to form its combat effectiveness. During this stage, the spares needed for equipment operation and repair are defined as initial spares; contractor needs to deliver it to army while the new equipment is deployed. During the process of equipment procurement, military personnel and contractor will negotiate together to determine the initial spares type and quantity, that is the so-called initial spares configuration problem.[21]

4.1. Support effectiveness evaluation targets of initial spares project

Before optimizing the initial spares configuration project, support effectiveness targets should be introduced, which are the criteria to evaluate spares project, and mainly contain effectiveness and economic target. The system effectiveness targets include the expected backorders of spares, equipment availability, spares fill rate, supply delay time, etc. Backorders is the spares shortages occurring at certain time, denoted as $B(X|s)$, X indicates the number of spares to be received, and s expresses the spares stock.

$$B(X|s) = \begin{cases} X-s, & X > s \\ 0, & X \leqslant s \end{cases} \qquad (4)$$

Expected backorders indicates the mean shortages of spares at any given time, denoted as EBO.

$$\mathrm{EBO} = p(X=s+1) + p(X=s+2) + \cdots + kp(X=s+k) + \cdots = \sum_{x=s+1}^{\infty}(x-s)p(X=x) \tag{5}$$

Suppose the expected backorders of item j at site m are denoted as EBO_{mj}, then we can get

$$\mathrm{EBO}_{mj} = \sum_{x_{mj}=s_{mj}+1}^{\infty}(x_{mj}-s_{mj})p(X_{mj}=x_{mj}) \tag{6}$$

where s_{mj} is the inventory of item j at site m and $p(X_{mj})$ the probability distribution of spares to be received. When variance to mean rate $\mathrm{Var}[X_{mj}]/E[X_{mj}] = 1$, $p(X_{mj})$ obeys Poisson probability distribution, if $\mathrm{Var}[X_{mj}]/E[X_{mj}] > 1$, $p(X_{mj})$ obeys negative binomial distribution, when $\mathrm{Var}[X_{mj}]/E[X_{mj}] < 1$, $p(X_{mj})$ obeys binomial distribution.

Equipment availability is related to its working time and none working time. From different perspectives, it is distinguished as operational availability A_o, supply availability A_s and inherent availability A_i. For single equipment, it is defined as the percentage of working hours at a given interval; for the group equipment, it indicates the proportion of equipment being in good state.[22] Different definitions about equipment availability are shown as follows.

$$A_\mathrm{o} = \frac{\mathrm{MTBF}}{\mathrm{MTBF}+\mathrm{MTTR}+\mathrm{MLDT}} \times 100\% \tag{7}$$

$$A_\mathrm{s} = \frac{\mathrm{MTBF}}{\mathrm{MTBF}+\mathrm{MLDT}} \times 100\% \tag{8}$$

$$A_\mathrm{i} = \frac{\mathrm{MTBF}}{\mathrm{MTBF}+\mathrm{MTTR}} \times 100\% \tag{9}$$

where MTBF is the mean time between failures, MTTR the mean time to repair and MLDT the mean logistic delay time. According to Eqs. (7)–(9), another formation about A_o can be defined as

$$A_\mathrm{o} = \frac{A_\mathrm{s}A_\mathrm{i}}{A_\mathrm{s}+A_\mathrm{i}-A_\mathrm{s}A_\mathrm{i}} \times 100\% \tag{10}$$

Under the replacement repair policy, equipment availability relies on the LRU's expected backorder; for series system, any LRU' shortage will result in system breakdown, so the spares supply availability at location m is given by

$$A_\mathrm{s}(m) = \prod_{j\in \mathrm{Inden}(1)}[1-\mathrm{EBO}_{mj}/(N_m Z_j)]^{Z_j} \tag{11}$$

where $A_\mathrm{s}(m)$ is the equipment spares supply availability at site m.

Spares fill rate, also known as support probability, refers to the probability that there are no spares shortages when it is to be required. This target is an effectiveness parameter related to spares inventory, so the filling rate for a single item can be expressed as

$$\mathrm{EFR}_{mj} = p(x_{mj} \leqslant s_{mj}-1) = \sum_{x_{mj}=0}^{s_{mj}-1} p(X_{mj}=x_{mj}) \tag{12}$$

Spares fill rate for equipment system EFR_m is

$$\mathrm{EFR}_m = \sum_{j\in \mathrm{Inden}(1)}\lambda_{mj}\mathrm{EFR}_{mj} \Big/ \sum_{j\in \mathrm{Inden}(1)}\lambda_{mj} \tag{13}$$

Spares supply delays, also known as support delay time, refers to delays for equipment repair time caused by lack of the necessary spares. For a single item, it is defined as the ratio of the spares' expected backorders to its average demand rate during the given time. The mean support delay time T_dm for system can be calculated as

$$T_\mathrm{dm} = \sum_{j\in \mathrm{Inden}(1)}\mathrm{EBO}_{mj} \Big/ \sum_{j\in \mathrm{Inden}(1)}\lambda_{mj} \tag{14}$$

4.2. Determination of spares maintenance supply turnover

To evaluate the effectiveness of initial spares project, we need to calculate the spares maintenance supply turnover. It is also called the spares pipeline, that is, the number of spares to be received. It consists of three parts: the first is the spares waiting for supply, the second is the spares being repaired, and the third is the spares being delayed for repaired. Suppose the supplier of site m is denoted as n, $n = \mathrm{SUP}(m)$, the items waiting for replacement can be derived from the number of backorders for item j at the supplier n; only a fraction f_{mj} of these backorders are destined for location m, and this fraction can be calculated as the ratio between the demand for item j at the supplier n: $f_{mj} = \lambda_{mj}\mathrm{NRTS}_{mj}/\lambda_{\mathrm{SUP}(m),j}$. So for this part of pipeline, the mean value is given as $f_{mj}\mathrm{EBO}_{\mathrm{SUP}(m)}$. The total number of items waiting for replacement of subassembly $k \in \mathrm{Sub}(j)$ equals the number of backorders BO_{mk}; only a fraction h_{mjk} of the backorders for item k at location m is due to a request from item j. A reasonable approximation for this fraction is the effective demand rate for subassembly k arising from item j as a fraction of the total demand rate for item k at location m. According to the conclusions of Ref.[20], we can get the mean value of pipeline for item j at the site m.

$$E[X_{mj}] = \lambda_{mj}(1-\mathrm{NRTS}_{mj})T_{mj} + \lambda_{mj}\mathrm{NRTS}_{mj}O_{mj} + f_{mj}\mathrm{EBO}_{\mathrm{SUP}(m),j} + \sum_{k\in \mathrm{Sub}(j)} h_{mjk}\mathrm{EBO}_{mk} \tag{15}$$

where T_{mj} means the average repair time of item j at site m, O_{mj} the jth item delivery time, NRTS_{mj} the probability that item j cannot be repaired at location m, during the process of repairing the item j, it will produce the demand for its subassembly k, the ratio to its total demand of item k is denoted as h_{mjk}. According to Eq. (15), we can get the variance of pipeline for item j at location m.

$$\mathrm{Var}[X_{mj}] = \lambda_{mj}(1-\mathrm{NRTS}_{mj})T_{mj} + \lambda_{mj}\mathrm{NRTS}_{mj}O_{mj} + f_{mj}(1-f_{mj})\mathrm{EBO}_{\mathrm{SUP}(m),j} + f_{mj}^2\mathrm{VBO}_{\mathrm{SUP}(m),j} + \sum_{k\in \mathrm{Sub}(j)} h_{mjk}^2\mathrm{VBO}_{mk} + \sum_{k\in \mathrm{Sub}(j)} h_{mjk}(1-h_{mjk})\mathrm{EBO}_{mk} \tag{16}$$

where the variance of backorders is denoted as VBO.

4.3. Optimization algorithm for initial spares configuration model

The optimization principle is that the spares cost should be minimized to ensure that the system effectiveness satisfies the target constraints. The initial spares optimization model is shown as

$$\begin{cases} \min & \sum_j \sum_m C_j s_{mj} \\ \mathrm{s.t} & A \geqslant A_0,\ \mathrm{EFR} \geqslant \mathrm{EFR}_0,\ T_\mathrm{d} \leqslant T_{\mathrm{d}0} \end{cases} \tag{17}$$

where C_j is the price of item j, s_{mj} the initial spares stock.

For initial spares optimization, we can introduce marginal algorithm, which is a gradual optimization technology that rationally controls the resources utilization through analyzing effectiveness vs cost of marginal units.[1] Under certain constraints, marginal algorithm will calculate repeatedly until meeting the target value, and during the computation process, the optimal control variables need to be determined. The basic steps of this algorithm are shown as follows.

Step 1. Initialize the spares stock, set $s_{mj} = 0$.

Step 2. Evaluate the system effectiveness and calculate equipment availability A_0, spares fill rate EFR_0, as well as support delays time T_{d0}.

Step 3. Operate iterative calculation, and during each iteration, calculate the marginal value $\delta_{mj}\,(s_{mj})$ of each spare in the whole support system, the calculation method is given as

$$\delta_{mj}(s_{mj}) = \frac{P_{mj}(s_{mj}+1) - P_{mj}(s_{mj})}{C_j} \tag{18}$$

Step 4. Compare each value of $\delta_{mj}\,(s_{mj})$ to determine the item and site that are most in need to adjust; the maximum marginal value denotes max (δ_{mj}), its corresponding optimal item is denoted as j^* and optimal site is denoted as m^*; make the item j^* at site m^* plus 1, and keep others unchanged.

Step 5. Judge whether the current system effectiveness can meet the setting target. If yes, the algorithm calculation ends and get the optimal spares project; otherwise, go to Step 3 and continue to calculate until meeting the target.

5. Procurement project of incomplete repairable items

For recoverable items, all of them can be supposed to be repaired after failure. However, for items that have limited service life, or items that their working performance may deteriorate as the increasing of repair time, it will be a certain discarding probability during equipment life cycle. As a result, spares stock state in the support system inventory will change; then, spares procurement needs to be considered. Inside the support system, ($s-1$, s) inventory replenishment policies are still to be suitable, but outside the support system, the top-echelon support site needs to purchase spares from external supplier. Taking into account that the external procurement and transportation costs are higher than internal ordering fee, spares reorder point and order quantity need to be determined.

5.1. Spares backorders function with the approximate Laplace demand distribution

First, spares demand probability distribution during supply cycle (or procurement delay) needs to be given and it is generally approximated to normal distribution. However, reorder point R is a nonlinear function about order quantity Q, we need constantly calculation to determine the optimal estimation for R and Q,[17] which will make the calculation process becomes tedious. For repairable items with low consumption, its interval demand can be approximated by Laplace distribution, which is shown in Fig. 2. In this way, we can not only ensure the accuracy of calculation result, but also make Q and R independent so that the calculation process will become easier.

Laplace demand probability distribution for incomplete repairable items during procurement delay time can be expressed as

$$p(x_{0j}) = (\sqrt{2}/2\sigma_{0j}) \exp\left(-\frac{\sqrt{2}|x_{0j} - E[D_{0j}]|}{\sigma_{0j}}\right) \tag{19}$$

where x_{0j} indicates consumption amount of item j, σ_{0j} the interval demand standard deviation of item j and $E[D_{0j}]$ the interval demand rate of item j; subscript symbol 0 indicates the top-echelon site in support system.

Suppose discarding probability of item j at top-echelon site is denoted as d_{0j}, and procurement cycle, which means the interval from orders issued to receive spares (also known as the order delays), is denoted as TD_j. If external suppliers have adequate spares, then, the expected demand rate of spares during procurement interval is

$$E[D_{0j}] = \lambda_{0j} d_{0j} \mathrm{TD}_j \tag{20}$$

where λ_{0j} indicates the average annual demand rate of item j at top-echelon site. Suppose the safety stock coefficient is denoted as k_{0j}, so the spares reorder point R_{0j} can be expressed as

$$R_{0j} = k_{0j}\sigma_{0j} + E[D_{0j}] \tag{21}$$

Here, Standard deviation σ_{0j} and expected interval demand $E[D_{0j}]$ are given parameters; if the value of k_{0j} is determined, optimal reorder point R_{0j} can be obtained. During the replenishment cycle, spares stock state changes between R_{0j} and $R_{0j} + Q_{0j}$ and probability distribution of spares backorders can be expressed as

$$p(\mathrm{BO}_{0j} = y) = \frac{1}{Q_{0j}} \int_{R_{0j}}^{R_{0j}+Q_{0j}} \frac{1}{\sqrt{2}\sigma_{0j}} \exp\left(\frac{-\sqrt{2}(L + y - E[D_{0j}])}{\sigma_{0j}}\right) \mathrm{d}L \tag{22}$$

where L expresses the stock state of item j, BO_{0j} is the backorders of item j at top-echelon. Expected backorders of item j at top-echelon site is shown as

$$\mathrm{EBO}_{0j} = \int_0^\infty y p(\mathrm{BO}_{0j} = y)\mathrm{d}y = \frac{\sigma_{0j}^2}{4Q_{0j}} \exp\left(-\sqrt{2}k_{0j}\left(1 - \mathrm{e}^{-\sqrt{2}Q_{0j}/\sigma_{0j}}\right)\right) \tag{23}$$

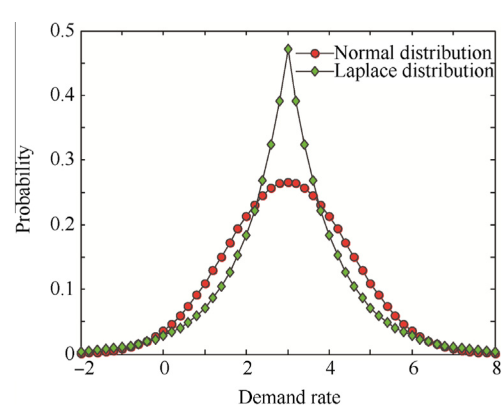

Fig. 2 Comparison of normal distribution and Laplace distribution.

5.2. Solution of spares procurement project

Under the precondition of satisfying the backorders constraint target, we will calculate the optimal reorder point R^* and order quantity Q^* to minimize the spares annual inventory management cost and order fees and the procurement optimization model is established as follows:

$$\begin{aligned}&\min \frac{\Omega_{0j}\lambda_{0j}d_{0j}}{Q_{0j}} + c'_{0j}\left[\frac{Q_{0j}+1}{2} + R_{0j} - E[D_{0j}] + \mathrm{EBO}_{0j}\right] \\ &\text{s.t.}\mathrm{EBO}_{0j} \leqslant B_{0j}, Q_{0j} \geqslant 1, R_{0j} \geqslant -1\end{aligned} \tag{24}$$

where Ω_{0j} means the fixed ordering cost of item j, c'_{0j} the annual inventory management cost for item j at top-echelon site and B_{0j} the backorders constraint target of item j. Bringing the constraint target into Eq. (24), establishing the Lagrange function and making partial derivation about R_{0j} and Q_{0j}, then, making Eq. (24) equal to 0, we can obtain

$$Q^*_{0j} = \frac{\sigma_{0j}}{\sqrt{2}} + \sqrt{\frac{2\Omega_{0j}\lambda_{0j}d_{0j}}{c'_{0j}} + \frac{\sigma_{0j}}{2}} \tag{25}$$

$$R^*_{0j} = -\frac{\sigma_{0j}}{\sqrt{2}} \ln \frac{4Q_{0j}B_{0j}}{\sigma^2_{0j}\left(1 - \mathrm{e}^{-\sqrt{2}Q_{0j}/\sigma_{0j}}\right)} + E[D_{0j}] \tag{26}$$

The optimal initial spares configuration project is denoted as s^*, and its expected backorders of item j is denoted as $\mathrm{EBO}_{0j}(s^*)$. Through analysis, we know that among the spares total demand, a fraction can be resolved through repairing, and the rest demand caused by spares consumption must be purchased from external supplier. Therefore, the excepted backorders constraint target B_{0j} needs to be allocated reasonably between two aspects; the allocation method is given below.

$$B_{0j} = \mathrm{EBO}_{0j}(s^*)\frac{E[D_{0j}]}{E[D_{0j}] + \lambda_{0j}(1 - d_{0j})T_{0j}} \tag{27}$$

where T_{0j} means the average repair time of item j at top-echelon site. According to inventory balance equations and based on Eq. (26), we must consider the average number of failure items being repaired at top-echelon site, as well as the number of items being replenished from top-echelon site to its downstream site l ($l \in \mathrm{Unit}(m)$). Then, we get

$$\begin{aligned}R^*_{0j} = &-\frac{\sigma_{0j}}{\sqrt{2}} \ln \frac{4Q_{0j}B_{0j}}{\sigma^2_{0j}\left(1 - \mathrm{e}^{-\sqrt{2}Q_{0j}/\sigma_{0j}}\right)} + E[D_{0j}] \\ &+ \lambda_{0j}(1 - d_{0j})T_{0j} - \sum_{l\in\mathrm{Unit}(0)} \lambda_{lj}\mathrm{NRTS}_{lj}O_{lj}\end{aligned} \tag{28}$$

5.3. Optimization process

The optimization analysis process for incomplete repairable items is shown in Fig. 3. At first, collect equipment reliability design information and determine its mission and operational conditions, and then forecast spares demand rate. Secondly, with initial spares model and algorithm, optimize its stock and then we can get the initial configuration project. Thirdly, evaluate support effectiveness of initial spares project and determine the support constraint targets. Finally, calculate spares reorder point and order quantity based on procurement model. According to the above steps, we can get the spares support project, including the initial spares configuration and its replenishment policy.

6. Application

Here we take a certain type of weapon control equipment as an example. It is equipped on four warships, which are denoted as No. 1, No. 2, No. 3, and No. 4. Spares of this equipment can be stored at warship stock for equipment's routine maintenance and temporary repair; besides, it can also be stored at support base warehouse as the turnover spares inventory. When spares stock on the warship cannot satisfy requirement of equipment repair or happens to be of shortage, base warehouse will supply its turnover spares to the warship. As spares consumption, the base warehouse will purchase spares from external supplier based on its inventory state so as to ensure the balance of spares inventory in the support system.

6.1. Determination of support parameters

(1) According to spares optimization process, the first work is to determine the equipment spares bill, which is shown in Table 2. It includes several important parameters, such as equipment structure codes, spares item, reliability target (MTBF), installation number of the jth item in its higher indenture component Z_j, duty cycle, discard rate, and the spares price. In the list, the equipment system is decomposed into the smallest replaceable units. There are 87 spares in this equipment. Considering the length of this paper is limited, here we list only a part of important spares, among which, the central computer is a large machine components, so it can be treated as

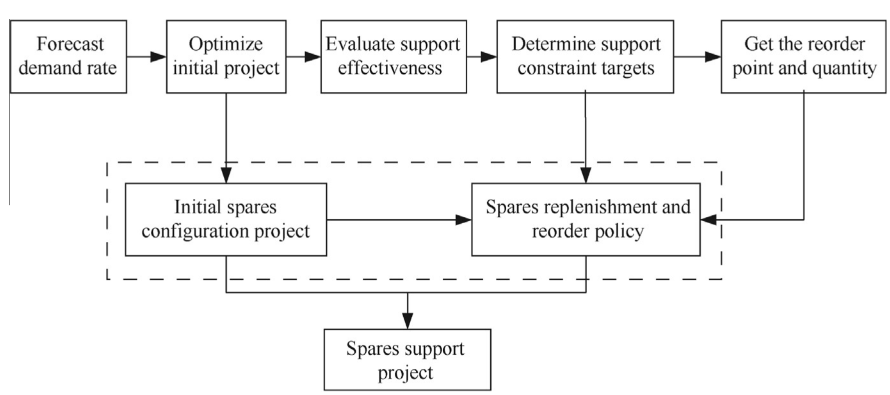

Fig. 3 Optimization and analysis process for incomplete repairable items.

Table 2 Equipment spares bill.

Spare	Property	MTBF	Z_j	Duty cycle	Discard rate	Price (Yuan)
Central computer			1	1		
Solver	LRU	900	1	1	0.1	56000
Display and control unit	LRU	910	1	1	0.1	48000
Bus board	LRU	100	1	1	0.1	10800
Interface board	LRU	1000	1	0.5	0.1	5400
Potentiometer	LRU	1200	1	0.5	1	1452
Toroidal inductor	SRU	1200	2	1	1	180
Transformer	LRU	1700	1	1	1	568
Transform plugin	LRU	238	1	1	0.1	17499
Module	SRU	900	2	0.9	1	1850
101 Instrument	LRU	2000	2	0.5	0.03	100000
75-II Instrument	LRU	200	1	1	0.03	2000
MS Instrument	LRU	200	1	1	0.03	20000
ZC-63	LRU	170	4	0.8	1	5000
Distribution box	LRU		1	1		10000

logical item, that is, it will not be treated as spares. In addition, there will be no failure for distribution box in its entire service cycle, which is also called as whole life components. Therefore, the distribution box can also be regarded as logic components.

(2) Determine the failure items repairable probability in the repair shop of warship and base. For the items that cannot be repaired after failure, the repairable probability equals 0. For the repairable items, because of the limitations of testing device, repair tools, personnel skills as well as technical information, the repair work on warship is generally confined to the regular maintenance, fault detection, removal and replacement for some key independent replaceable units (LRU) in the equipment. Suppose the LRU's repairable probability on the warship equals 0.5, and the SRU's repairable probability equals 0. The base repair shop generally has a strong repair capability; therefore, the failure items, without considering its discard and consumption, can be always repaired in base repair shop, so its repairable probability equals 1.

(3) Determine the repair time of failure items. Large statistics show that the repair time of failure items at warship and base approximates to 0.5 and 4 days, respectively.

(4) Determine the transshipment delays and supply cycle. Suppose the transshipment delays (delivery period) from base to warship equals 1 day and the supply cycle from external supplier to base equals 10 days.

(5) Determine spares inventory management costs and ordering fees. Inside the support system, spares ordering fees can be neglected. Outside the support system, the fixed ordering fees from base to external supplier are 100 dollars; the inventory management fee rate (proportion of spares inventory management costs account to its price) is 0.05.

(6) Determine equipment deployment and its operational intensity. Suppose that the equipment installation amount on each warship equals 1, and its average weekly working time is 7 h on warship No. 1, 8 h on warship No. 2, 10 h on warship No. 3, and 8 h on warship No. 4, respectively.

Optimization of spares project is mainly measured between support effectiveness and costs; therefore, optimization results under the two kinds of circumstances are shown in this part: the one is the equipment availability constraint project, the other is the support cost constraint project.

6.2. Scenario 1: availability constraint optimization

Suppose the equipment availability target is required no less than 0.95, namely $A \geqslant 0.95$. Optimal curves of effectiveness vs cost for initial spares project is shown in Fig. 4.

When satisfying the setting equipment availability target, we calculate and get the initial spares configuration and its procurement optimization project (see Table 3). Under this project, the equipment availability $A = 0.951$, spares fill rate EFR = 0.826, support delays $T_{\rm d} = 5.1$ h, and the spares total investment $C = 767950$ Yuan.

In Table 3, if reorder point equals -1, it indicates that spares need to be ordered from external supplier when its shortage occurs once.

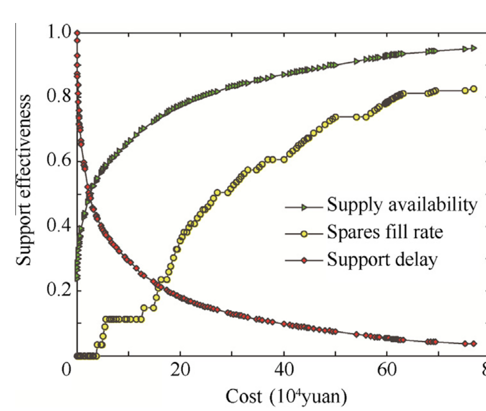

Fig. 4 Optimal curves of effectiveness vs cost for availability constraint optimization project.

Table 3 Optimal spares project under availability constraint.

Spare	No. 1 warship	No. 2 warship	No. 3 warship	No. 4 warship	Base site	Reorder point	Reorder quantity
Solver	0	0	0	0	1	0	1
Display and control unit	0	0	0	0	1	0	1
Bus board	1	1	1	1	1	0	1
Interface board	0	0	0	0	1	0	1
Potentiometer	1	1	1	1	1	0	2
Toroidal inductor	0	0	0	0	2	1	7
Transformer	0	0	0	0	2	1	7
Transform plugin	0	0	1	0	1	0	1
Module	0	0	0	0	1	0	3
101 Instrument	0	0	0	0	0	−1	1
75-II Instrument	1	1	1	1	1	0	1
MS Instrument	0	0	0	0	1	0	1
ZC-63	1	1	1	1	2	1	1
Distribution box	0	0	0	0	0	−1	1

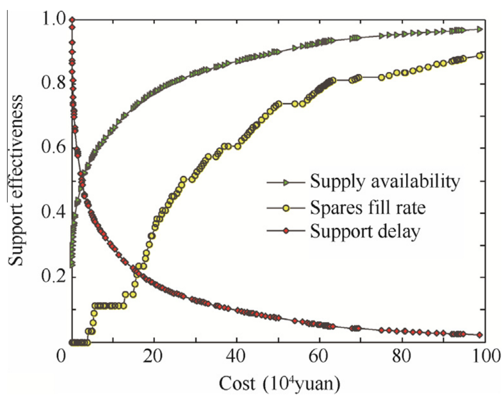

Fig. 5 Optimal curves of effectiveness vs cost for support cost constraint optimization project.

6.3. Scenario 2: cost constraint optimization

Suppose the target of spares budget cannot exceed 100 million, the optimal curves of effectiveness vs cost is shown in Fig. 5.

When satisfying the support cost target constraint, the initial spares configuration and its procurement optimization project are shown in Table 4. Under this project, the expected equipment availability $A = 0.968$, spares fill rate EFR = 0.887, support delay $T_{\mathrm{d}} = 3.26$ h, and the total investment for spares procurement $C = 987247$ Yuan.

6.4. Optimization result analysis

With the analysis of spares optimization results under the two scenarios, we get some conclusions below.

(1) For independent replaceable units (LRU) that have characteristics of high failure rate or low price, they should be stored in the warship stock.
(2) The shop replacement units (SRU) cannot be repaired and replaced on the warship, so there are no spares of SRU in the warship stock.
(3) At the base support site, repairing the failure unit LRU will consume its sub-assembly SRU; therefore, for the SRU of high price and low failure rate, its stock at base site is low and the procurement quantity from external supplier is very small. For the SRU of low price and high failure rate, we should keep several stocks at base site, and its procurement quantity from external supplier is large. For example, the ring inductance, its initial configuration at the base stock is 2, and the reorder quantity from external supplier is 7.

Table 4 Optimal spares project under support cost constraint.

Spare	No. 1 warship	No. 2 warship	No. 3 warship	No. 4 warship	Base site	Reorder point	Reorder quantity
Solver	0	0	0	0	1	0	1
Display and control unit	0	0	0	0	1	0	1
Bus board	1	1	1	1	1	0	1
Interface board	0	1	1	1	1	0	1
Potentiometer	1	1	1	1	1	0	2
Toroidal inductor	0	0	0	0	2	1	7
Transformer	0	0	0	0	2	1	7
Transform plugin	1	1	1	1	1	0	1
Module	0	0	0	0	1	0	3
101 Instrument	0	0	0	0	0	−1	1
75-II Instrument	1	2	2	2	1	0	1
MS Instrument	1	1	1	1	1	0	1
ZC-63	1	1	1	1	2	1	1
Distribution box	0	0	0	0	0	−1	1

(4) For the LRU of high price and low failure rate, it should be stored in base stock.
(5) For the expensive spares with quite low failure rate, both warship and base site keep zero stock. Usually, such spares will not be consumed during its life cycle. For example, for the 101 Instrument priced 100000, the mean time between failure is 2000 h, and its average annual operation time is just only 300–400 h; analysis reveals that the failure occur once every 5 to 6 years. Therefore, it belongs to a typical strategic support resource, usually stocked in the naval strategic reserve warehouse.

Qualitative analysis helps to get the conclusion that the model optimization results are consistent with actual case, so at the qualitative level, the modeling correctness, optimization method rationality and the project feasibility are verified.

Under the two scenarios, we can get different spares configuration projects, and the result of support effectiveness evaluation will be also different, which is shown in Fig. 6, where the values of support delay and total cost are normalized.

During spares optimization, we only consider the impact factor of spares supply; therefore, the equipment availability only expresses the supply availability. In order to get the operational availability, we must consider some other factors, such as items failure detection, spares replacement or installation.

Take the Scenario 1 for example, after optimization, the supply availability $A_s = 0.951$. Equipment repair at warship mainly includes the failure detection, spares replacement and installation. Suppose the mean time to repair MTTR = 6 h, and MTBF = 250 h. According to Eq. (9), we can get the inherent availability A_i.

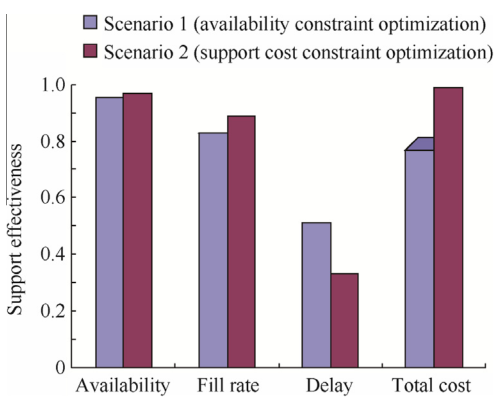

Fig. 6 Difference of spares project under two scenarios.

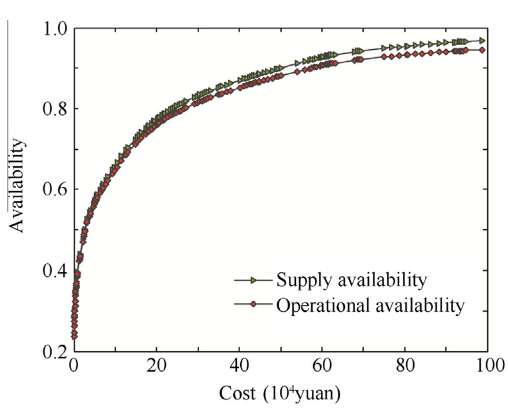

Fig. 7 Relationship between A_s and A_o.

$$A_i = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}} = \frac{250}{250 + 6} = 97.7\%$$

Then, combining the Eq. (11), operational availability can be calculated.

$$A_o = \frac{A_s A_i}{A_s + A_i - A_s A_i} = \frac{0.951 \times 0.977}{0.951 + 0.977 - 0.951 \times 0.977} = 93\%$$

It is obvious from above analysis that even if there are enough spares in stock, we can still not make the equipment operational availability reach 100%. Fig. 7 shows the relationship between A_s and A_o.

If there are adequate spares in stock, spares supply availability A_s can be approximated to 100%, then, $A_o = A_i = 0.977$.

7. Conclusions

(1) Configuration and optimization method of partial repairable spares is developed in this paper. Considering the spares' degradation after repair as well as its discarding rate, we establish a configuration model through organic combination of $(s-1, s)$ and (R, Q) inventory policy so as to make the calculation result reasonable.
(2) In the section of application research, two scenarios are given: one is availability constraint optimization project and the other is cost constraint optimization project. In addition, we compare the two scenarios results; with the help of configuration results analysis, some relative conclusions are obtained, which show that the results mainly depend on equipment reliability, task intensity and support cost. In addition, it may be also closely related to other factors, such as repair conditions, spares property and support organization structure.
(3) According to the research in this paper, we get initial spares configuration project as well as spares procurement project, and the modeling method reflects the ideas of system and integrated optimization. Conclusions in this paper are significant for equipment integrated logistics support: at the stage of equipment design, it can provide theory and methods for spares determination; at the stage of equipment in service, it can provide effective approaches for spares support project optimization, which is useful for military personnel.
(4) Generally, initial logistics support data are provided by equipment design and research department. These data are mainly generated from experience accumulated during the process of reliability test. When equipment is armed in forces, it will also be influenced by equipment working environment, task intensity, support mode, etc. So there may be some differences between initial and actual support information. Therefore, we should constitute maintenance support information standards and establish equipment support database.

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